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Control Stick Logic in High-Angle-of-Attack Maneuvering

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The relationships between pilot control stick inputs and control effector deflections are examined. Specifically, we address multiply redundant control effector arrangements and command-driven control laws. During high-angle-of-attack, low-dynamic-pressure maneuvering, there is both a control power and control coordination problem. Control effector deflections are not one to one with pilot inputs, and the maximum capabilities of effectors to respond to pilot inputs varies dynamically with the state of the airplane. The problem is analyzed in the context of a generic control law that continuously regulates sideslip. A means is presented to relate the fixed control effector limits to the dynamically varying control response limits. This information may be used to re-establish the one-to-one correspondence of pilot inputs to control capabilities.

Nomenclature

F	= force
G	= gearing ratio
I	= moment of inertia
L	= transformation matrix
L, D, C	= aerodynamic lift, drag, and sideforce, wind axes
L, M, N	= rolling, pitching, and yawing moments
m	= mass
$\mathbf{m}$	= vector of moments (or moment coefficients)
p, q, r	= rolling, pitching, and yawing rates
T	= thrust
$\mathbf{u}$	= vector of control effectors
V	= velocity
α	= angle of attack
β	= sideslip angle
δ	= pilot control input
θ, ϕ	= Euler angles

Subscripts

A	= bare-airframe aerodynamic force or moment
B	= body axes
C	= control-generated force or moment
c	= commanded
l	= lower
p	= principal axes
u	= upper
W	= wind axes

Introduction

IN the past few years there has been an increased emphasis on exploiting the capabilities of high-performance tactical airplanes to maneuver at high angles of attack and low dynamic pressure. This emphasis has resulted in rapid advances in several areas of dynamics and control, including the design of nonlinear flight control laws[1–5] and the utilization of multiple control effectors.[3,6–9]

There is seldom a problem with control power or control coordination when maneuvering at high speeds. Conventional aerodynamic control effectors are capable of creating angular accelerations in excess of the limitations of the pilot and in some cases of the airframe. At the low angles of attack associated with high-speed flight, the airplane's principal body axes and nearly aligned with the wind axes. As a result, the pilot's control actions are very nearly kinematically decoupled in roll, pitch, and yaw, so that the control inputs required to maneuver are intuitive.

Low-speed, high-angle-of-attack maneuvering places demands on both control power and control coordination. At low dynamic pressure, many of the controls are only marginally effective, and maneuvering capability is determined by the physical limitations of these controls. At high angles of attack, the wind and reference body axis systems are far from coincident, so that lateral-directional control and response becomes highly coupled. The control coordination must be provided either by the pilot or the flight control system.

The control law answer to the low-dynamic-pressure/high-angle-of-attack coordination problem has been to translate the pilot's stick and rudder inputs into demands for angular accelerations in the appropriate coordinate system and to drive the control effectors as required. Multiaxis maneuvering at the limits of the airplane's capabilities using multiply redundant control effectors, usually requires a complex control allocation scheme.

With the control law interpreting the pilot's inputs and working out the details of control allocation, correspondence between the pilot's control inputs and the actual control effector deflections becomes very complicated. To illustrate this, consider a simple full lateral stick input at high angle of attack. The stick input may be interpreted as a requirement for a first-order roll response about the velocity vector, accompanied by no change in either angle of attack or sideslip. This is the classic velocity vector roll. The angular accelerations required to accomplish this maneuver are quite complex and generally require moments about all three axes. This in turn requires carefully blended combinations of the actual control effectors, and the right combination will vary throughout the maneuver. The actual deflections of the control effectors at any point in time are clearly not one to one with the pilot's inputs.

Past research in relating the pilot's demands to the control effector limits has treated maximum demands as invariant and then attempted to find the combination of control effector deflections that "best" satisfied that demand. This approach is reflected in the design of modern tactical aircraft[4] in which, for example, lateral control sensitivity is expressed as a number ($P_{ss}/\delta_{s,lat}$) that is fixed over quite large ranges of dynamic pressure. Thus full lateral stick is interpreted as a demand for a roll response that results (through an artificially generated roll mode time constant) in the desired steady-state roll rate. Half lateral deflection is interpreted as half the maximum steady-state roll rate. If a given demand is not attainable, then various schemes may be used to best satisfy the demand. In early control allocation research, we considered the control effectors' moment-generating capabilities and elected to scale the excess moment demands to the maximum attainable moment in the same direction (as a vector in moment space) as that demanded. There is little justification for doing so, save that it is relatively easy to calculate. More defensible is the notion of allocating the controls so as to minimize the difference between the airplane's actual trajectory and that which would result if the controls were unconstrained.

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During any period of time in which a control law is making excessive demands of the controls, the control law is invalid: The assumptions on which it is based are no longer true. The actual trajectory of the airplane that results is not predicted by any simple theory, and to minimize the difference between the actual and desired trajectories may be a meaningless exercise. If the actual trajectory results in a departure from controlled flight, then our minimization has only picked the smallest elephant in a charging herd to be the one to trample us.

We propose to resolve this problem by relating the maximum deflections of the pilot's controls to the maximum effects attainable at the current flight condition. Any effect assigned to a particular pilot input—sideslip to rudder deflection, for example—may be analyzed at some level as a demand for the combinations of control-generated moments required to achieve the effect. These demanded moments must lie within the attainable moment-generating capabilities of the control effectors. We determine the control effector coordination required to generate a particular effect and the maximum effect that the coordinated controls can physically attain. The results are the dynamically varying maximum capabilities of the control law/control effectors. This information may then be used in the interpretation of the pilot's inputs to ensure that the full capabilities of the airplane are available to the pilot and that the pilot never commands more performance than the airplane is capable of delivering.

Analysis

Assumptions

We make the usual assumptions of a flat, nonrotating, inertial Earth and a rigid body. The notation used below is that according to Etkin in Ref. 10. Our analysis will be conducted in both the wind-axis and a body-fixed-axis system. We assume for simplicity that all necessary transformations have been made so that our reference body-axis system coincides with the principal axis system. The airplane is assumed to have a plane of symmetry.

The major assumption below is that the desired and actual sideslip β is zero. Nonzero sideslip could be accommodated in the analysis with some increase in complexity, but it would not qualitatively change the results. Our objective is to determine the conditions under which an ideal control law operating with limitations on the available control effectors can achieve the goal of zero sideslip. We also take the component of thrust in the y direction (wind or body axes) to be zero. We assume that the aerodynamic side force is a function only of sideslip and control deflections; with our assumption of zero sideslip, the aerodynamic side force is therefore a function of control deflection alone.

We assume that the aerodynamic moments may be affinely separated into bare-airframe moments (subscript A) plus contributions from the controls (subscript C):

$$
\begin{aligned}
L &= L_A + L_C \\
M &= M_W = M_A + M_C \\
N &= N_A + N_C
\end{aligned}
\tag{1}
$$

We will require knowledge of the moment-generating capabilities of the control effectors. From this we assume the moments that are attainable within the constraints imposed on the effectors are known and can be analyzed.

Equations of Motion

The force equations in the wind-axis system are given by

$$
\begin{aligned}
F_{xw} &= m\dot{V} \\
F_{yw} &= mVr_w \\
F_{zw} &= -mVq_w
\end{aligned}
\tag{2}
$$

where

$$
\begin{aligned}
F_{xw} &= T_{xw} - D - mg\sin\theta_w \\
F_{yw} &= T_{yw} - C + mg\cos\theta_w\sin\phi_w \\
F_{zw} &= T_{zw} - L + mg\cos\theta_w\cos\phi_w
\end{aligned}
\tag{3}
$$

The body-axis pitching moment equation, with the assumption of principal axes, is

$$
M = I_{yp}\dot{q} - (I_{zp} - I_{xp})rp \tag{4}
$$

The pilot's lateral-directional inputs will be related to wind-axis angular accelerations, for which the equations of motion are

$$
\begin{aligned}
L_W &= \dot{I}_x p_W - \dot{I}_{xz} r_W + I_x \dot{p}_W - I_{xz}\dot{r}_W \\
&\quad + (I_z - I_y) q_W r_W - I_{xz} p_W q_W
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
N_W &= \dot{I}_z r_W - \dot{I}_{xz} p_W + I_z \dot{r}_W - I_{xz}\dot{p}_W \\
&\quad + (I_y - I_x) p_W q_W + I_{xz} q_W r_W
\end{aligned}
\tag{6}
$$

The body-axis forces, moment, and angular rates are related to their counterparts in the wind axes by the transformation L_{BW}. With our assumption of zero sideslip, this transformation is

$$
L_{BW} = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix} \tag{7}
$$

where α is measured from the reference (principal) body x axis to the wind x axis.

The instantaneous values of the moments of inertia in the wind axes are related to the principal moments of inertia by

$$
\begin{bmatrix} I_x & 0 & -I_{xz} \\ 0 & I_y & 0 \\ -I_{xz} & 0 & I_z \end{bmatrix} = L_{WB} \begin{bmatrix} I_{xp} & 0 & 0 \\ 0 & I_{yp} & 0 \\ 0 & 0 & I_{zp} \end{bmatrix} L_{BW}
$$

or

$$
\begin{aligned}
I_x &= I_{xp}\cos^2\alpha + I_{zp}\sin^2\alpha \\
I_y &= I_{yp} \\
I_z &= I_{xp}\sin^2\alpha + I_{zp}\cos^2\alpha \\
I_{xz} &= (I_{xp} - I_{zp})\sin\alpha\cos\alpha
\end{aligned}
\tag{8}
$$

The inertia rates needed to Eqs. (5) and (6) are found by differentiating Eqs. (8):

$$
\begin{aligned}
\dot{I}_x &= 2(I_{zp} - I_{xp})\sin\alpha\cos\alpha\dot{\alpha} \\
\dot{I}_z &= -2(I_{zp} - I_{xp})\sin\alpha\cos\alpha\dot{\alpha} \\
\dot{I}_{xz} &= (I_{xp} - I_{zp})(\cos^2\alpha - \sin^2\alpha)\dot{\alpha}
\end{aligned}
\tag{9}
$$

Pilot Inputs

Rudder Pedals

Rudder pedal inputs are taken as commands for sideslip. The means to effect the commands is through the application of appropriate wind-axis yawing moments. The primary aerodynamic effects of sideslip occur in the rolling and yawing moments and in the side force C.

The wind-axis yaw rate is given by the second of Eqs. (2). The total side force is $F_{yw} = T_{yw} - C + mg\cos\theta_w\sin\phi_w$. For convenience we neglect the component of thrust in the y direction and assume the aerodynamic side force is a function only of the control effector deflections $\boldsymbol{u}$ and the sideslip β. We further assume that none of the control effectors has the capability of generating pure side force and that this effect is secondary to their moment-generating capabilities.

Further assume that the control law has done its job perfectly, and the actual sideslip is equal to that desired, here assumed to be zero. Denote by C^* the aerodynamic side force that is present when the sideslip is its commanded value $\beta_c = 0$. Then, for arbitrary values of thrust, orientation of the wind axes, and control effector positions, there is a corresponding F^*_{yW} and a yaw rate r^*_W that must be present if these conditions are to hold:

$$
r^*_W = \frac{F^*_{yW}(\beta_c, \boldsymbol{u}, \phi_W, \theta_W)}{mV} \tag{10}
$$

This exact value of r_W^* must be continuously generated by our perfect control law, or the actual value of sideslip will not track that which is desired. This is accomplished by the generation of appropriate rolling and yawing moments, Eqs. (5) and (6), for which we require $\dot{r}_W^*$:

$$\dot{r}_W^* = \frac{\dot{F}_{yw}^*}{mV} - \frac{F_{yw}^*\dot{V}}{mV^2} = \frac{\dot{F}_{yw}^*}{mV} - \frac{F_{yw}^*F_{xw}}{m^2V^2} \tag{11}$$

Denote by L_W^* and N_W^* the wind-axis rolling and yawing moments that obtain when $r_W = r_W^*$ and $\dot{r}_W = \dot{r}_W^*$:

$$L_W^* = \dot{I}_x p_W - \dot{I}_{xz} r_W^* + I_x \dot{p}_W - I_{xz}\dot{r}_W^* + (I_z - I_y) q_W r_W^* - I_{xz} p_W q_W \tag{12}$$

$$N_W^* = \dot{I}_z r_W^* - \dot{I}_{xz} p_W + I_z \dot{r}_W^* - I_{xz}\dot{p}_W + (I_y - I_x) p_W q_W + I_{xz} q_W r_W^* \tag{13}$$

The moments L_W^* and N_W^* consist of bare-airframe aerodynamic contributions and of control effector contributions. The control effector contributions are therefore those that our perfect control law must have commanded in order to regulate the sideslip. Any other lateral-directional control requirements must be satisfied in such a manner that the required wind-axis yaw acceleration is unaffected.

Lateral Stick

Lateral stick inputs are taken as commands for roll rates about the instantaneous velocity vector. The velocity vector is not fixed with respect to Earth nor with respect to the body axes, except that (with our assumption of no sideslip) it always lies in the plane of symmetry of the airplane.

The wind-axis roll rate is to be commanded by the pilot's input, according to a gearing ratio G_p that relates maximum lateral stick to some maximum wind-axis roll rate. Denote the commanded input as p_{W_c}, so that

$$p_{W_c} = G_p \delta_p \tag{14}$$

We assume that our perfect control law shapes the rolling response according to a first-order response,

$$\dot{p}_{W_c} = \lambda_p \left(p_W - p_{W_c}\right) \tag{15}$$

Longitudinal Stick

There are several choices for the interpretation of longitudinal stick inputs. Here we consider longitudinal stick inputs as either angle-of-attack or body-axis pitch rate commands. Neither interpretation is without difficulties. For high-angle-of-attack maneuvering, a body-axis pitch rate command system suffers since, with $\beta = 0$, we have $\dot{\alpha} = q - q_W$. The commanded parameter is q, and q_W is determined by the third of Eqs. (2). Thus the possibility exists for α to attain unacceptable values. On the other hand, pilots (excluding engineering test pilots trying to get a data point) are not particularly interested in the specific value of angle of attack, only that it contribute to the accomplishment of some task. Moreover, there may be no logical choices for minimum and maximum desired angle of attack that we can relate to full forward or aft stick. At high dynamic pressures, angle of attack has clear meaning to the pilot as a load factor command, but there is no such simple relationship in the regions of highly nonlinear lift vs angle-of-attack of concern here.

We assume the airplane has no direct lift control capability and that any change in angle of attack will be commanded through control of the body-axis pitch rate. With this assumption, both the angle-of-attack and body-axis pitch rate command systems will command some specific value of $\dot{q}$. For simplicity, we assume a first-order response similar to that used for lateral commands:

$$q_c = G_q \delta_q \tag{16}$$

$$\dot{q}_c = \lambda_q (q - q_c) \tag{17}$$

The commanded variables are therefore β, p_W, and q. We will designate the pilot's inputs as δ_β, δ_p, and δ_q, with stick and rudder limits of ± 1 unit from centered positions.

Control Effector Requirements

The capabilities of the control effectors to generate moments may be represented in any desired coordinate system, but such data are normally available in the reference body-axis system. We therefore incorporate the interpreted pilot inputs into the wind-axis rolling and yawing moment equations, and project the requirements back into the reference body axes. After some simplification, we arrive at

$$L^* = I_{xp}\lambda_p \cos\alpha\left(p_W - p_{W_c}\right) + \frac{F_z \sin\alpha}{mV}(I_{yp} - I_{xp})p_W + \frac{F_y \cos\alpha}{mV}(I_{zp} - I_{xp})q + (I_{zp} - I_{xp})\sin\alpha(p_W q) + \frac{F_y F_z \cos\alpha}{m^2V^2}(I_{yp} - I_{xp}) + \frac{F_x F_y \sin\alpha}{m^2V^2}\dot{I}_{xp} - \frac{\dot{F}_y \sin\alpha}{mV}I_{xp} \tag{18}$$

$$M^* = I_{yp}\lambda_q (q - q_c) + \frac{F_y(I_{xp} - I_{zp})\cos 2\alpha}{mV}p_W + \frac{1}{2}\sin 2\alpha (I_{xp} - I_{zp})p_W^2 - \frac{F_y^2(I_{xp} - I_{zp})\sin 2\alpha}{2m^2V^2} \tag{19}$$

$$N^* = I_{zp}\lambda_p \sin\alpha\left(p_W - p_{W_c}\right) + \frac{F_z \cos\alpha}{mV}(I_{zp} - I_{yp})p_W + \frac{F_y \sin\alpha}{mV}(I_{zp} - I_{xp})q + (I_{zp} - I_{yp})\cos\alpha(p_W q) + \frac{F_y F_z \sin\alpha}{m^2V^2}(I_{yp} - I_{zp}) - \frac{F_x F_y \cos\alpha}{m^2V^2}\dot{I}_{zp} - \frac{\dot{F}_y \cos\alpha}{mV}I_{zp} \tag{20}$$

Recall that, built into these equations, are the yaw rates and accelerations required to maintain zero sideslip. The moment contributions of the control effectors that are required to maintain zero sideslip are

$$L_c^* = L^* - L_A(p, r, \beta) \tag{21}$$

$$M_c^* = M^* - M_A(q, \alpha, \dot{\alpha}) \tag{22}$$

$$N_c^* = N^* - N_A(p, r, \beta) \tag{23}$$

The control effector requirements are related to the pilot's commands as

$$L_c^* = -I_{xp}\lambda_p \cos\alpha p_{W_c} + f_L(\alpha, \beta, V, p_W, \phi_W, \theta_W) \tag{24}$$

$$M_c^* = -I_{yp}\lambda_q q_c + f_M(\alpha, \beta, q, V, p_W, \phi_W, \theta_W) \tag{25}$$

$$N_c^* = -I_{zp}\lambda_p \sin\alpha p_{W_c} + f_N(\alpha, \beta, V, p_W, \phi_W, \theta_W) \tag{26}$$

The functions f_L, f_M, and f_N represent everything that is left over in Eqs. (21–23) after separating the commanded variables. These functions are therefore the moments the controls have to generate to satisfy the requirement $\beta = 0$, given the current values of p_W and q, and the other states listed as arguments.

We may use Eqs. (24) and (26) to characterize the roll–yaw coordination required of the control effectors in response to a lateral stick command. At any instant, we must have

$$\frac{\Delta N_c}{\Delta L_c} = \frac{-I_{zp}\lambda_p \sin\alpha \Delta p_{W_c}}{-I_{xp}\lambda_p \cos\alpha \Delta p_{W_c}} = \frac{I_{zp}}{I_{xp}}\tan\alpha \tag{27}$$

Pitch commands, of course, are orthogonal to the roll–yaw plane. This fact, along with the requirement of Eq. (27), defines a plane surface in three-dimensional space of the moment-generating capabilities of the control effectors. This plane is affine to the origin of the control effectors' moment space and must pass through a point

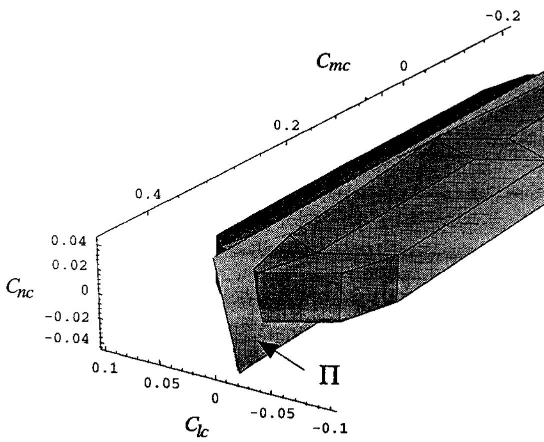

Fig. 1 Attainable moment subset.

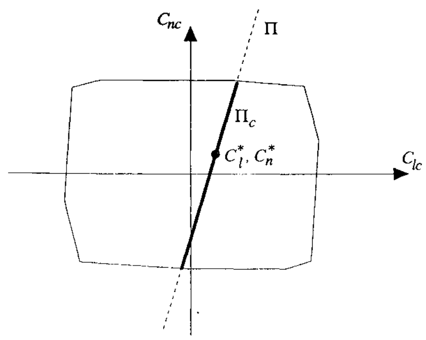

Fig. 2 Π_c plane of $C_m = C_m^*$.

defined by Eqs. (24–26). We will denote this plane as Π. The pilot may command any combination of wind-axis rolling and body-axis pitching accelerations in this plane without creating unwanted yawing accelerations.

Attainable Moments

Associated with any configuration of multiply redundant control effectors that are constrained to certain limits is a set of moments the effectors may generate within those constraints. This is the attainable moment subset (AMS) and has been fully described in Refs. 8 and 9. Depending on the scheme used to allocate the control effectors, the actual attainable moments may be less than the maximum AMS. To avoid having to address the various control allocation schemes in use, we assume that the maximum AMS is available.

In the subsequent discussion, we will deal with the moment coefficients rather than the moments themselves. Figure 1 shows a typical AMS intersected by the plane Π defined by Eq. (27). Although the AMS depicted is based on actual data for a seven-control F-18 fighter configuration, we will treat it as representative of multiply redundant control configured tactical airplanes. The moments shown in Fig. 1 are, of course, deltas to be applied to the bare-airframe C_l, C_m, and C_n to yield the total moments.

The plane Π must intersect the AMS or the airplane is instantaneously out of control. If this plane does not intersect the AMS, then the moments required to satisfy the pilot's commands and to regulate the sideslip are not available, and for some finite time these variables will evolve according to whatever moments are available. The position and orientation of Π are dynamically varying, so it is possible that the states of the airplane will evolve in such a manner that the airplane returns to a controlled condition. The ultimate consequences of such a condition are not predictable, but for some finite time the airplane has departed.

Denote the intersection of Π with the AMS as Π_c, and assume $\Pi_c \neq \emptyset$. Figure 2 shows Π and Π_c in the plane $C_m = C_m^*$. The orientation of Π with respect to the C_{lc}–C_{nc} axes is determined by Eq. (27) and here is representative of an airplane with $I_x : I_z = 1 : 7$, $\alpha = 30$ deg. Also shown in the figure is a notional point C_l^*, C_n^* from Eqs. (18) and (20). At the instant at which Fig. 2 is valid, the pilot may command wind-axis roll rates that correspond to any point on Π_c.

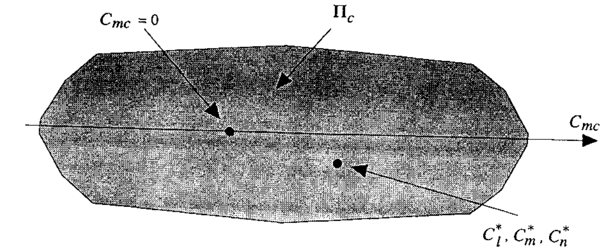

Fig. 3 In-plane view of Π_c.

Figure 3 is a notional view of Π_c from a normal to its surface.

Interpreting the Pilot's Inputs

At first glance it appears we must evaluate the functions f_L, f_M, and f_N in Eqs. (24–26) before we can determine the location of (C_l^*, C_m^*, C_n^*) and hence the moment-generating capabilities that are left over to satisfy the pilot's inputs. Indeed, many of the terms in these functions are routinely assumed known in the design of dynamic inversion and model-following control laws.

Rather than evaluate the functions, we can infer the location of (C_l^*, C_m^*, C_n^*) by returning to the assumption of a near-perfect control law. This control law correctly calculates the control effector deflections required to maintain zero sideslip, but it does not "know" the minimum and maximum values of p_W and q that it is allowed to command within the constraints imposed on the control effectors.

If the control law assumes too great a value for these extremes, then the control effectors will be saturated before the pilot's stick inputs have reached their maximum deflections. In other words, a small nose-down pitch command could exhaust the nose-down pitching moment capabilities of the airplane. A large nose-down pitch command would produce exactly the same response as a small one. Pilots naturally expect that more stick will yield more response and are likely to find this characteristic unsatisfactory in terms of flying qualities.

A more insidious condition arises if the control law assumes too small a value for the extremes of p_W and q that it may command. The control law would then not be utilizing the maximum capabilities of the control effectors. The unused capabilities translate directly into unnecessary extra weight on the airplane: smaller or fewer control effectors, properly utilized, could obtain the same performance as the existing ones. Seen another way, the pilot executing a defensive maneuver against a surface-to-air missile wants maximum performance from the airplane, not just that which was convenient to design and still meet specifications.

Let us then assume the control law has some fixed values in mind for the maximum values of p_W and q that it is allowed to command, say $G_p\delta_p$ and $G_q\delta_q$. The actual value is not important, since it is bound to be wrong anyway. To avoid cluttering the discussion, let us take a pure right lateral stick input (longitudinal stick fixed) of arbitrary magnitude for consideration, or $p_{W_c} = xG_p\delta_p$, $0 < x < 1$. The control law calculates the control effector deflections necessary, according to $\dot{p}_{W_c} = \lambda_p(p_W - p_{W_c})$. Since the control law is also regulating the sideslip, this combination of controls lies in Π by definition. If we know the effectiveness of the control effectors (loosely, the B matrix) at the current flight condition, then C_{lc}^*, C_{mc}^*, and C_{nc}^* are known. The orientation of Π is completely determined by specifying I_x, I_z, and α in Eq. (27), here presumed known. We therefore know the position and orientation of Π relative to the original of the AMS, and that is sufficient for our purposes. Thus Fig. 2 is determined, and the point C_l^*, C_n^* is the point our control law has selected based on a maximum p_{W_c} of $G_p\delta_p$.

To continue, we must assume the AMS is known and that we may calculate the intersections of arbitrary lines with the facets of the AMS. Methods for determining the AMS and performing the necessary calculations were developed in Ref. 9. We may therefore calculate the boundaries of Π_c on the AMS. Figure 4 shows the two points of interest, denoted $\boldsymbol{m}_u$ and $\boldsymbol{m}_l$. For a right stick deflection, $\boldsymbol{m}_u$ is the moment of interest.

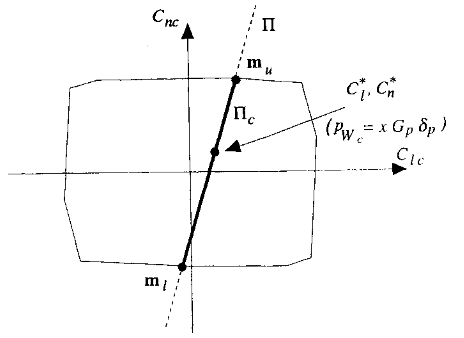

Fig. 4 Upper and lower attainable moments in Π_c.

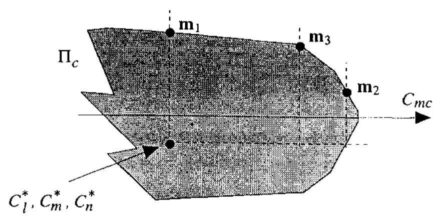

Fig. 5 Conflicting roll–pitch requirements.

The rest is algebra. Since we know the moments at some percentage of our presumed maximum and at the actual (as yet unknown) maximum, we may write

$$L_{cu} = -I_{xp}\lambda_p \cos\alpha\left(p_W - p_{W_{c,\max}}\right) + f_L \tag{28}$$

$$L_c^* = -I_{xp}\lambda_p \cos\alpha(p_W - xG_p\delta_p) + f_L \tag{29}$$

so that

$$p_{W_{c,\max}} = \frac{L_c^* - L_{cu}}{I_{xp}\lambda_p \cos\alpha} + xG_p\delta_p \tag{30}$$

Here we have used the rolling moment equation to calculate the actual maximum commandable roll rate $p_{W_{c,\max}}$. The yawing moment equation would do as well, unless α is very small and $\sin\alpha$ nears zero. The yawing moment should, of course, be used if α nears 90 deg.

Note that the minimum (maximum negative, or left) roll rate that may be commanded is in general not the negative of the maximum positive roll rate. This is caused by the irregular shape of the AMS plus the overhead of maintaining zero sideslip.

We may obviously repeat the procedure described above for a longitudinal pilot input with the lateral command fixed. At any instant we can calculate the minimum and maximum rolling and pitching rates that the pilot may command. We may elect to intercept the control law's commands and modify them to reflect this knowledge, so that at each instant the pilot has the maximum of the airplane's capabilities at his fingertips. Alternatively, if the minima and maxima thus determined are slowly varying, we may simply update the control stick gearing for use in the next iteration.

We have been careful to consider the lateral and longitudinal inputs independently in the preceding discussion. The reason for this is that there will almost always be a conflict between maximum pitching and rolling requirements. This is obvious in view of the fact that Fig. 3 is not a rectangle. The physical origin of this problem lies in the use of opposing (left–right) control effectors both differentially and symmetrically. Consider, for example, the horizontal tail surfaces. If they are generating a maximum pitching moment, then there is no possible differential movement for rolling. If they are generating a maximum rolling moment, then their symmetric contribution is their average deflection, which is now fixed.

Figure 5 shows the rolling–pitching conflict schematically. This stick is assumed to be full aft and full right.

In Fig. 5, the point m_1 represents the effect of satisfying the lateral command first: There is nothing left over for the longitudinal command. Point m_2 represents the effect of satisfying the longitudinal command first and using what is left over for the lateral command. Point m_2 may be considered slightly preferable to m_1, since there is at least a small amount of roll response.

Along the boundary of Π_c from m_1 to m_2 are other solutions, for example m_3. If the pilot is willing to give up some pitch command to get more roll, or vice-versa, there may be some solution like m_3 that the pilot prefers to all the other choices. The problem is stick position alone does not signal such a preference: the stick is in the corner. The pilot does not have a picture of Fig. 5, which at any rate is dynamically varying.

We leave the question of roll–pitch conflict resolution with two suggested remedies. Neither remedy is conventional, but we offer them with a straight face to a community that has seen two-position wings (F-8 Crusader) and telescoping control sticks (early A-4 Skyhawks).

1) Since stick position, once at its limits, does not convey the pilot's intentions or desires, we suggest that the addition of stick force measurements will. Thus, even though the stick is in the corner, the pilot will presumably be trying to bend the stick more in one direction than another. If this bending is in the direction of, say, m_3 in Fig. 5, then m_3 is used to apportion the rolling and pitching commands.

2) The problem with the surface Π_c is that its shape does not match the rectangular shape of the control stick limits. It is unlikely that any combination of multiply redundant control effectors could make Π_c rectangular. We therefore propose that a stick feel system impose artificial limits on the control stick travel such that it is shaped like Π_c. If this can be done, then the one-to-one correspondence between the pilot's inputs and the airplane's responses will be established, and the airplane will have no surprises or hidden capabilities for the pilot.

No-Sideslip Requirement

It is obvious that no control law can perfectly regulate sideslip at all times. The equations derived above for the no-sideslip requirement do not constitute a control law but only state what moments a perfect control law must be commanding if it is perfectly regulating sideslip. A practical control law will not be able to anticipate those moment requirements, but instead will be responding to measured nonzero sideslip.

The response of a control law to such errors in regulation will generally be to command a wind-axis yaw acceleration to drive the nonzero sideslip toward zero. Such an acceleration is orthogonal to Π_c. We may therefore think of Π_c not as a mathematical plane, but rather as having some thickness. The thickness of Π_c then represents the potential control activity required to regulate the sideslip and will depend upon how sideslip regulation is performed, i.e., what the gains are and how much sideslip can be reasonably expected during maneuvering.

Conclusions

We have presented an analysis of the interrelationships between pilot's inputs, the control law's interpretation of those inputs, and the control effectors' ability to satisfy the control law's demands. The path from the stick to the control effector is convoluted indeed and admits of no obvious generalizations or simplifications.

A method was presented to calculate the instantaneous interpretation of stick inputs in terms of the maximum commendable wind-axis roll rate and pitch rate. Clearly the most demanding requirement of the calculation is the knowledge of the attainable moment-generating capabilities of the control effectors. That such knowledge is required should come as no surprise, however, since the limitations of the control effectors lie at the heart of the problem.

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